
DOI: 10.59461/ijitra.v2i3.70

The online version of this article can be found at: https://www.ijitra.com/index.php/ijitra/issue/archive

Published by:
PRISMA Publications

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Inventory model for Exponential Deterioration Items with Power Dependent Demand

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ABSTRACT

In this paper we develop and analyze an inventory model assumption that deterioration rate follows Exponential distributions with power dependent demand. With shortage and without shortage both cases have been taken care of in developing the inventory models. Shortages are fully backlogged whenever they are allowed. Through numerical examples the results are illustrated. The sensitivity analysis for the model has been performed to study the effect changes of the values of the parameters associated with the model.

Keywords: EOQ model; deteriorating items; shortage; power dependent demand; Exponential distribution

I. INTRODUCTION

The influence and maintenance inventories for deteriorating items with shortages have received much attention of several researchers in the recent years because most of the physical goods deteriorate over period of time. In real life, many of the items are either damaged or decayed or affected by some other factors and is not in a perfect condition to satisfy the demand. Food items, drugs, pharmaceuticals, radioactive substances are examples of such items where deterioration can take place during the normal storage period of the commodity and consequently this loss must be taken into account when analyzing the system. So decay or deterioration of physical goods in stock is a very realistic feature and researchers felt the necessity to use this factor into consideration in developing inventory models.

Ghare and Schrader (1963) who developed an economic order quantity model with constant rate of decay. An order-level inventory model for a system with constant rate of deterioration have proposed by Shah and Jaiswal (1977), Aggarwal [1978], Dave and Patel [1981]. Inventory models with a time dependent rate of deterioration were developed by Covert and Philip [1973], Mishra [1973] and Deb and Chaudhuri [1986]. Some of the significant recent work in this area have been done by Chung and Ting [1993], Fujiwara [1993], Hariga [1996], Hariga and Benkerouf[1994], Wee [1995], Jalan et al. [1999], Su, et al. [1996], Chakraborty and Chaudhuri [1997], Giri and Chaudhuri[1997], Chakraborty, et al. [1997] and Jalan and Chaudhuri, [1999], etc.
At the beginning, demand rate were assumed to be constant which is in general likely to be time dependent and stock dependent. Begum et al. [2010] have developed economic lot size model for price dependent demand. Inventory model for ameliorating items for price dependent demand was proposed by Mondal et al. [2003], with the motivation of C. K. Tripathy, et al. [2010] and Sushil Kumar, et al. [2013] we developed EOQ models for Weibull deteriorating items and price dependent demand.

In this paper, we have developed generalized EOQ model for deteriorating items where deterioration rate follows two parameter Weibull and demand rate is considered to be a function of selling price. For the model where shortages are allowed they are completely backlogged. Here we have considered both the case of with shortage and without shortage in developing the model. Using differential equations, the profit rate function are obtained. By maximizing the profit rate function, the optimal production schedule and optimal production quantity are derived. Through numerical illustration the sensitivity analysis is carried. This model also includes some of the earlier models as particular cases for particular or limiting values of the parameters.

II. ASSUMPTIONS AND NOTATIONS

The following assumptions are made for developing the model:

a) The demand rate is a function of selling price which is \( f(s) = (a - bs) > 0 \)

b) Shortages, whenever allowed are completely backlogged.

c) The deterioration rate is proportional to time.

d) Replenishment is instantaneous and lead time is zero.

e) \( T \) is the length of the cycle.

f) \( Q \): Ordering quantity in one cycle

g) \( a \): Ordering cost

h) \( C \): Cost per unit

i) \( h \): Inventory holding cost per unit per unit time

j) \( p \): Shortages cost per unit per unit time

k) \( s \): Selling price per unit and

l) The deterioration of units follows the two parameter Weibull distribution with probability density function \( f(t) = \alpha \beta t^{\beta-1} e^{-\alpha t^\beta}, \quad 0 < \alpha < 1 \) is scale parameter and \( \beta > 0 \) is shape parameter and \( t > 0 \). Therefore, the instantaneous rate of replenishment is \( \alpha \beta t^{\beta-1} \)

m) During time \( t_1 \), inventory is depleted due to deterioration and demand of the item. At time \( t_1 \) the inventory becomes zero and shortages start occurring.

III. MATHEMATICAL FORMULATION OF THE MODEL

Let \( I(t) \) be the inventory level at time ‘t’ \( (0 \leq t \leq T) \). The differential equations governing the system in the cycle time \([0, T]\) are

\[
\frac{d}{dt} I(t) + \alpha \beta t^{\beta-1} I(t) = -(a - bs) \quad 0 \leq t \leq t_1
\]

\[
\frac{d}{dt} I(t) = -(a - bs) \quad t_1 \leq t \leq T
\]

Solving the equations (1) and (2) and neglecting higher powers of \( a \), we get

\[
I(t) = \frac{(a - bs)}{\alpha t^{\beta}} \left( (t_1 - t) + \frac{a}{\beta + 1} (t_1^{\beta+1} - t^{\beta+1}) \right) \quad 0 \leq t \leq t_1
\]

\[
I(t) = (a - bs)(t - t_1) \quad t_1 \leq t \leq T
\]

Stock loss due to deterioration in the cycle of length \( T \) is

\[
L(T) = (a - bs) \int_{0}^{t_1} e^{ax^\beta} dt - (a - bs) \int_{0}^{t_1} dt
\]

\[
= (a - bs) \left[ \frac{ax^{\beta+1}}{\beta+1} \right]_{0}^{t_1}
\]

Ordering quantity \( Q \) in the cycle of length \( T \) is

\[
Q = L(T) + \int_{0}^{t} (a - bs) dt
\]
\[ H = h \left( \int_{0}^{t_1} l(t) dt \right) = h \left[ \int_{0}^{t_1} \frac{(a - bs)}{e^{at\beta}} \left( (t_1 - t) + \frac{\alpha}{\beta + 1} (t_1^{\beta + 1} - t^{\beta + 1}) \right) dt \right] \]

Holding cost is obtained by substituting the equations (3) and (4), we get

\[ H = h(a - bs) \left( t_1 - \frac{at_1^{\beta + 1}}{\beta + 1} \left( t_1 + \frac{at_1^{\beta + 1}}{\beta + 1} \right) - \int_{0}^{t_1} te^{-at\beta} dt + \frac{\alpha}{\beta + 1} \int_{t_1}^{T} t^{\beta + 1} e^{-at\beta} dt \right) \]

Neglecting higher powers of \( \alpha \), we get

\[ H = - \int_{t_1}^{T} te^{-at\beta} dt + \frac{\alpha}{\beta + 1} \int_{t_1}^{T} t^{\beta + 1} e^{-at\beta} dt \]  

(7)

Shortage cost during the cycle is

\[ S = - \int_{t_1}^{T} l(t) dt = - \int_{t_1}^{T} (a - bs)(t_1 - t) dt \]

\[ = \frac{1}{2} (a - bs)(T - t_1)^2 \]  

(8)

Let \( P(T, t_1, s) \) be the profit rate function. Since the profit rate function is the total revenue per unit minus total cost per unit time, we have

\[ P(T, t_1, s) = s(a - bs) - \frac{1}{T} \left( A + C \left( \frac{a}{\beta + 1} t_1^{\beta + 1} + (a - bs)T \right) \right) \]

(9)

Substituting the values of equations (6), (7) and (8) in equation (9), one can get the profit rate function as

\[ P(T, t_1, s) = s(a - bs) - \frac{1}{T} \left( A + C(a - bs) \left( \frac{a\gamma T^{\beta + 1}}{\beta + 1} + T \right) \right) \]

\[ + h(a - bs) \left[ \gamma T^2 - \left( \frac{a\gamma T^{\beta + 1} T^{\beta + 2}}{\beta + 1} \right) - \int_{0}^{T} te^{-at\beta} dt + \frac{\alpha}{\beta + 1} \int_{0}^{T} t^{\beta + 1} e^{-at\beta} dt \right] \]

\[ + \frac{\alpha}{2} (a - bs)(T - t_1)^2 \]  

(10)

Let \( t_1 = \gamma T, 0 < \gamma < 1 \)

Hence we get the profit function

\[ P(T, s) = s(a - bs) - \frac{1}{T} \left( A + C(a - bs) \left( \frac{a\gamma T^{\beta + 1}}{\beta + 1} + T \right) \right) \]

\[ + h(a - bs) \left[ (\gamma T^2) - \left( \frac{a\gamma T^{\beta + 1} T^{\beta + 2}}{\beta + 1} \right) - \int_{0}^{T} te^{-at\beta} dt + \frac{\alpha}{\beta + 1} \int_{0}^{T} t^{\beta + 1} e^{-at\beta} dt \right] \]

\[ + \frac{\alpha}{2} (a - bs)(T - \gamma T)^2 \]  

(11)

Our objective is to maximize the profit function \( P(T, s) \). The necessary conditions for maximizing the profit function are

\[ \frac{\partial P(T, s)}{\partial \gamma} = 0 \quad \text{and} \quad \frac{\partial P(T, s)}{\partial s} = 0 \]

We get

\[ (a - bs) \left( \frac{Ca\gamma T^{\beta + 1} T^{\beta - 1}}{\beta + 1} + h \left( \frac{a^2 \gamma^2 T^{\beta + 2} (2\beta + 1) T^{\beta}}{(\beta + 1)^2} \right) \right) \]

\[ - \left[ \frac{1}{2T^2} \int_{0}^{T} te^{-at\beta} dt + \frac{\alpha}{\beta + 1} \int_{0}^{T} t^{\beta + 1} e^{-at\beta} dt \right] \]

\[ + \frac{\alpha}{T} \frac{\partial}{\partial T} \left[ \frac{1}{2} \int_{0}^{T} te^{-at\beta} dt + \frac{\alpha}{\beta + 1} \int_{0}^{T} t^{\beta + 1} e^{-at\beta} dt \right] + \frac{\alpha}{2} (1 - \gamma)^2 = 0 \]  

(12)
and
\[(a - bs) + \frac{b}{T} C \left(\frac{\alpha y^\beta + 1 T^{\beta+1}}{\beta + 1} + T\right) + h \left(\left(\alpha y^\beta + 1\right)^2 - \frac{\alpha y^\beta + 1}{\beta + 1} T^{2\beta+2}\right)\]
\[- \int_0^T t e^{-at} dt + \frac{\alpha}{\beta + 1} \int_0^T y^\beta + 1 T^{\beta+1} e^{-at} dt + \frac{\pi}{2} (1 - y)^2 = 0 \quad (13)\]

Using the software Matcad 15, we obtain the optimal policies of the inventory system under study. To find the optimal values of \(T\) and \(s\), we obtain the first order partial derivatives of \(P(T, s)\) given in equation (11) with respect to \(T\) and \(s\) and equate them to zero. The condition for maximization of \(P(T, s)\) is
\[D = \left| \begin{array}{cc}
\frac{\partial^2 P(T, s)}{\partial T^2} & \frac{\partial^2 P(T, s)}{\partial T \partial s} \\
\frac{\partial^2 P(T, s)}{\partial s^2} & \frac{\partial^2 P(T, s)}{\partial s^2}
\end{array} \right| < 0\]

### IV. NUMERICAL EXAMPLE

**Case – I (with shortages)**

Let \(A = 500, C = 10, h = 2, \pi = 0.5, \alpha = 10, \beta = 0.5, \gamma = 0.4, a = 100, b = 2\)

Based on above input data and Using the software Matcad 6.0, we calculate the optimal value of \(t_1^* = 1.1484, T^* = 2.871, s^* = 34.976, Q^* = 62.627, P^*(T, s) = 310.964\)

**Case – II (without shortages)**

Based on above input data and Using the software Matcad 6.0, we calculate the optimal value of \(t_1^* = 0.8084, T^* = 2.021, s^* = 18.39, Q^* = 120.43, P^*(T, s) = 166.199\)

### V. SENSITIVITY ANALYSIS

To study the effects of changes of the parameters on the optimal profit derived by proposed method, a sensitivity analysis is performed considering the numerical example given above. Sensitivity analysis is performed by changing (increasing or decreasing) the parameters by 10% and 20% and taking one parameter at a time, keeping the remaining parameters at their original values. The results are shown in Table-1 and Table-2 for with shortage case and without shortage case respectively. The relationship between the parameters and the optimal values are shown in Figure 1 and 2.

<table>
<thead>
<tr>
<th>Variation Parameters</th>
<th>Optimal Policies</th>
<th>Change in parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>-20%</td>
</tr>
<tr>
<td>a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>t^*_1</td>
<td>1.285</td>
<td>1.238</td>
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<tr>
<td>T^*</td>
<td>3.214</td>
<td>3.097</td>
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<tr>
<td>s^*</td>
<td>31.073</td>
<td>32.883</td>
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<tr>
<td>Q^*</td>
<td>35.161</td>
<td>52.005</td>
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<tr>
<td>P^*(T, s)</td>
<td>143.527</td>
<td>174.373</td>
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<tr>
<td>b</td>
<td></td>
<td></td>
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<tr>
<td>t^*_1</td>
<td>2.076</td>
<td>1.605</td>
</tr>
<tr>
<td>T^*</td>
<td>5.192</td>
<td>4.014</td>
</tr>
<tr>
<td>s^*</td>
<td>24.868</td>
<td>29.573</td>
</tr>
<tr>
<td>Q^*</td>
<td>99.519</td>
<td>88.286</td>
</tr>
<tr>
<td>P^*(T, s)</td>
<td>428.809</td>
<td>411.187</td>
</tr>
<tr>
<td>a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>t^*_1</td>
<td>0.867</td>
<td>0.867</td>
</tr>
<tr>
<td>T^*</td>
<td>2.168</td>
<td>2.169</td>
</tr>
<tr>
<td>s^*</td>
<td>30.119</td>
<td>30.615</td>
</tr>
<tr>
<td>Q^*</td>
<td>67.11</td>
<td>65.225</td>
</tr>
<tr>
<td>P^*(T, s)</td>
<td>350.938</td>
<td>345.726</td>
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<tr>
<td>b</td>
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<td></td>
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<tr>
<td>t^*_1</td>
<td>0.804</td>
<td>0.842</td>
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<tr>
<td>T^*</td>
<td>2.012</td>
<td>2.105</td>
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<tr>
<td>s^*</td>
<td>30.134</td>
<td>30.168</td>
</tr>
<tr>
<td>Q^*</td>
<td>66.023</td>
<td>65.719</td>
</tr>
<tr>
<td>P^*(T, s)</td>
<td>343.059</td>
<td>343.344</td>
</tr>
</tbody>
</table>
Fig 1: Relationship between parameters and optimal values with shortages

We study from above Table-1 reveals the following
i) Increase in the values of either of the parameters $a$, will result in increase of $T^*$, $s^*$ and $Q^*$ but decrease $t_1^*$, $P^*(T,s)$.
ii) Decrease in the values of either of the parameters $a$, will result in decrease of $T^*$, $s^*$ and $Q^*$ but increase $t_1^*$, $P^*(T,s)$.
iii) Increase in the values of either of the parameters $b$, will result in increase of $s^*$ but decrease $t_1^*$, $T^*$, $Q^*$ and $P^*(T,s)$.
iv) Decrease in the values of either of the parameters $b$, will result in decrease of $t_1^*$ and $s^*$ but increase $T^*$, $Q^*$ and $P^*(T,s)$.
v) Increase in the values of either of the parameters $\alpha$, will result in increase of $t_1^*$, $T^*$ and $s^*$ but decrease $Q^*$ and $P^*(T,s)$.
vi) Decrease in the values of either of the parameters $\alpha$, will result in decrease of $t_1^*$, $T^*$ and $s^*$ but increase $Q^*$ and $P^*(T,s)$.
vii) Increase in the values of either of the parameters $\beta$, will result in increase of $t_1^*$, $T^*$ and $s^*$ but decrease $Q^*$ and $P^*(T,s)$.
viii) Decrease in the values of either of the parameters $\beta$, will result in decrease of $t_1^*$, $T^*$ and $s^*$ but increase $Q^*$ and $P^*(T,s)$.
Table 2

Sensitivity analysis of the model (without shortages)

<table>
<thead>
<tr>
<th>Variation Parameters</th>
<th>Optimal Policies</th>
<th>Change in parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>t1*</td>
<td>-20%</td>
</tr>
<tr>
<td>a</td>
<td></td>
<td>0.770</td>
</tr>
<tr>
<td>T</td>
<td></td>
<td>1.541</td>
</tr>
<tr>
<td>s</td>
<td></td>
<td>17.675</td>
</tr>
<tr>
<td>Q</td>
<td></td>
<td>130.339</td>
</tr>
<tr>
<td>P*(T, s)</td>
<td>171.313</td>
<td>176.242</td>
</tr>
<tr>
<td>b</td>
<td></td>
<td>1.02</td>
</tr>
<tr>
<td>T</td>
<td></td>
<td>2.04</td>
</tr>
<tr>
<td>s</td>
<td></td>
<td>17.731</td>
</tr>
<tr>
<td>Q</td>
<td></td>
<td>130.015</td>
</tr>
<tr>
<td>P*(T, s)</td>
<td>182.2</td>
<td>184.828</td>
</tr>
<tr>
<td>α</td>
<td></td>
<td>1.538</td>
</tr>
<tr>
<td>t1*</td>
<td></td>
<td>3.076</td>
</tr>
<tr>
<td>T</td>
<td></td>
<td>18.039</td>
</tr>
<tr>
<td>s</td>
<td></td>
<td>198.7</td>
</tr>
<tr>
<td>Q</td>
<td></td>
<td>227.355</td>
</tr>
<tr>
<td>P*(T, s)</td>
<td>289.195</td>
<td>223.586</td>
</tr>
</tbody>
</table>

Legend:
- **a**: Percentage change in parameters
- **b**: Variations in T
- **α**: Variations in s
- **β**: Variations in Q

Graphs showing variations in t1*, T, s, and Q with respect to changes in parameters.
We study from above Table-2 reveals the following

i) Increase in the values of either of the parameters a, will result in increase of $t^*_1$, $T^*$, $s^*$, $Q^*$ and $P^*(T, s)$.

ii) Decrease in the values of either of the parameters a, will result in increase of $t^*_1$, $T^*$, $s^*$, $Q^*$ and $P^*(T, s)$.

iii) Increase in the values of either of the parameters b, will result in increase of $t^*_1$, $T^*$, $s^*$, $Q^*$ and $P^*(T, s)$.

iv) Decrease in the values of either of the parameters b, will result in increase of $t^*_1$, $T^*$, $s^*$, $Q^*$ and $P^*(T, s)$.

v) Increase in the values of either of the parameters $\alpha$, will result in decrease of $t^*_1$, $T^*$, $s^*$, $Q^*$ and $P^*(T, s)$.

vi) Decrease in the values of either of the parameters $\alpha$, will result in increase of $t^*_1$, $T^*$, $s^*$, $Q^*$ and $P^*(T, s)$.

vii) Increase in the values of either of the parameters $\beta$, will result in decrease of $t^*_1$, $T^*$, $s^*$, $Q^*$ and $P^*(T, s)$.

viii) Decrease in the values of either of the parameters $\beta$, will result in increase of $t^*_1$, $T^*$, $s^*$, $Q^*$ and $P^*(T, s)$.

VI. CONCLUSION

In this paper economic production quantity models are developed and analyzed for a single commodity under consideration. It is possible to develop EPQ models for multiple commodities using random production (variable rate of production). Throughout the thesis it is assumed that the money value remain constant over the period of time i.e. the inflation has no influence on the models. It is also possible to develop and analyze the EPQ models developed in this paper with inflation (time values of money) which require further investigation.

VII. REFERENCES


